Mr. Michael T. Davis Calculus	Sections 5.2 & 5.3 Practice Quiz April 14, 2017

Name: Solutions

1. Determine whether the function $f(x)=x^2+3x-10$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0,4]$. If it does, then find the value of c that satisfies the equation $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

f(x) is continuous on [0,4] and differentiable on (0,4)

secant slope $m=\dfrac{f(4)-f(0)}{4-0}=\dfrac{28}{4}=7$

$f'(x)=2x+3$ $\quad f'(c)=2c+3$ $\quad 2c+3=7$ $\quad c=2$

2. Determine whether the function $f(x)=\cos x$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0,2\pi]$. If it does, then find the value of c that satisfies the equation $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

f(x) is continuous on $[0,2\pi]$ and differentiable on $(0,2\pi)$

secant slope $m=\dfrac{f(2\pi)-f(0)}{2\pi-0}=\dfrac{1-1}{2\pi}=\dfrac{0}{2\pi}=0$

$f'(x)=-\sin x$ $\quad f'(c)=-\sin c$ $\quad 0=-\sin c$ $\quad \sin c=0$

$c=\pi$

3. Determine whether the function $f(x)=\dfrac{x}{x-1}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0,2]$. If it does, then find the value of c that satisfies the equation $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

f(x) is not continuous on [0,2] since f(1) = undefined

4. Determine whether the function $f(x)=x^{\frac{4}{3}}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1,8]$. If it does, then find the value of c that satisfies the equation $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

$f(x)$ is continuous on $[-1,8]$ $\quad f'(x)=\frac{4}{3}x^{\frac{1}{3}}$

$f(x)$ is differentiable on $(-1,8)$

secant slope $m=\frac{16-1}{8--1}=\frac{15}{9}=\frac{5}{3}$ $\quad f'(c)=\frac{4}{3}c^{\frac{1}{3}}$

$\frac{5}{3}=\frac{4}{3}c^{\frac{1}{3}}$ $\quad \frac{5}{4}=c^{1/3}$ $\quad c=\frac{125}{64}$

5. Determine whether the function $f(x)=x^{\frac{2}{3}}$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1,27]$. If it does, then find the value of c that satisfies the equation $f'(c)=\dfrac{f(b)-f(a)}{b-a}$.

$f(x)$ is continuous on $[-1,27]$ $\quad f'(x)=\frac{2}{3}x^{-\frac{1}{3}}$

$f'(x)=\frac{2}{3x^{1/3}}$ $\quad f(x)$ is not differentiable at $x=0$

6. Determine the interval(s) on which the function $f(x)=8x-x^2$ is (a) increasing and (b) decreasing. $\quad f'(x)=8-2x$ $\quad 0=8-2x$ $\quad x=4$

$+ + + + \quad - - - -$

$\longleftarrow\!\!-\!\!-\!\!\bullet\!\!-\!\!-\!\!\longrightarrow$ f'

4

$f(x)$ is increasing on $(-\infty,4)$ since $f'(x)>0$

$f(x)$ is decreasing on $(4,\infty)$ since $f'(x)<0$

7. Determine the interval(s) on which the function $f(x)=e^x$ is (a) increasing and (b) decreasing. $\quad f'(x)=e^x$ $\quad e^x\neq 0$ $\quad e^x>0$ on $(-\infty,\infty)$

$f(x)$ is increasing on $(-\infty,\infty)$

8. Determine the interval(s) on which the function $f(x)=e^{x^2-6x}$ is (a) increasing and (b) decreasing.

$f'(x) = (2x-6)\,e^{x^2-6x}$ $\quad f'(3)=0$

$- - - - \quad + + + + \quad f'$

3

$f(x)$ is increasing on $(3,\infty)$ since $f'(x)>0$

$f(x)$ is decreasing on $(-\infty,3)$ since $f'(x)<0$

9. Determine the interval(s) on which the function $f(x)=\dfrac{1}{x-3}$ is (a) increasing and (b) decreasing.

$f'(x) = \dfrac{-1}{(x-3)^2}$ $\quad \dfrac{-1}{(x-3)^2} \neq 0$ $\quad f'(3)=$ undefined

$- - - - \quad - - - - \quad f'$

3

$f(x)$ is never increasing.

$f(x)$ is decreasing on $(-\infty,3)\cup(3,-\infty)$ since $f'(x)<0$

10. Determine the interval(s) on which the function $f(x)=\dfrac{x^3}{3}+\dfrac{x^2}{2}-6x+5$ is (a) increasing and (b) decreasing.

$f'(x) = x^2+x-6$ $\quad f'(x)=0$

$0 = x^2+x-6$ $\quad 0=(x+3)(x-2)$ $\quad f'(-3)=0$ $\quad f'(2)=0$

$+ + + \quad - - - \quad + + + \quad f'$

$-3 \qquad 2$

$f(x)$ is increasing on $(-\infty,-3)\cup(2,\infty)$ since $f'(x)>0$

$f(x)$ is decreasing on $(-3,2)$ since $f'(x)<0$

11. Determine the interval(s) on which the function $f(x)=\dfrac{1}{x^2}$ is (a) increasing and (b) decreasing.

$f'(x) = \dfrac{-2}{x^3}$ $\quad f'(x)\neq 0$ $\quad f'(0)=$ undefined

$+ + + + + \quad - - - - - \quad f'$

0

$f(x)$ is increasing on $(-\infty,0)$ since $f'(x)>0$

$f(x)$ is decreasing on $(0,\infty)$ since $f'(x)<0$

12. Use the Concavity Test to determine the interval(s) on which the graph of the function $f(x)=x^3-3x^2-9x+1$ is (a) concave up and (b) concave down.

$f'(x) = 3x^2 - 6x - 9 \quad f''(x) = 6x - 6 \quad f''(1) = 0$

$- - - - \quad + + + + \quad f''$

1

$f(x)$ is concave up on $(1, \infty)$ since $f''(x) > 0$

$f(x)$ is concave down on $(-\infty, 1)$ since $f''(x) < 0$

13. Use the Concavity Test to determine the interval(s) on which the graph of the function $f(x)=x^4-12x^3+48x^2-x+2$ is (a) concave up and (b) concave down.

$f'(x) = 4x^3 - 36x^2 + 96x - 1 \quad f''(x) = 12x^2 - 72x + 96$

$0 = 12x^2 - 72x + 96 \quad 0 = x^2 - 6x + 8 \quad 0 = (x-2)(x-4)$

$f''(2) = 0 \quad f''(4) = 0$

$+ + + \quad - - - \quad + + + \quad f''$

$2 \quad 4$

$f(x)$ is concave up on $(-\infty, 2) \cup (4, \infty)$ since $f''(x) > 0$

$f(x)$ is concave down on $(2, 4)$ since $f''(x) < 0$

14. Find all the points of inflection of the function $f(x)=x^3+2x^2-x-2$

$f'(x) = 3x^2 + 4x - 1 \quad f''(x) = 6x + 4 \quad f''(x) = 0$

$0 = 6x + 4 \quad 6x = -4 \quad x = -\frac{2}{3}$

$- - - - \quad + + + + \quad f''$

$-\frac{2}{3}$

$f(x)$ has a point of inflection at $x = -\frac{2}{3}$ since $f(x)$ has a tangent line at $x = -\frac{2}{3}$ and $f''(x)$ changes sign at $x = -\frac{2}{3}$

15. Find all the points of inflection of the function $f(x)=xe^x$

$f'(x) = e^x + xe^x \quad f''(x) = e^x + e^x + xe^x = 2e^x + xe^x$

$0 = e^x(2+x) \quad f''(-2) = 0$

$- - - - - - \quad + + + + \quad f''$

-2

$f(x)$ has a point of inflection at $x = -2$ since $f(x)$ has a tangent at $x = -2$ and $f''(x)$ changes sign at $x = -2$

16. Find all the points of inflection of the function $f(x)=x^4-6x^2+12$

$f'(x) = 4x^3 - 12x \quad f''(x) = 12x^2 - 12 \quad 0 = 12x^2 - 12$

$0 = x^2 - 1 \quad f''(-1) = 0$ and $f''(1) = 0$

$+ + + \quad - - - \quad + + + \quad f''$

$-1 \quad 1$

$f(x)$ has points of inflection at $x = -1$ and $x = 1$ since $f(x)$ has tangents at $x = -1$ and $x = 1$ and since $f''(x)$ changes sign at $x = -1$ and $x = 1$